Zoom Class 5/5/2020

Integration Problems

Ques Evaluate $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$

Soln $I = \int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}} = \int_0^1 \frac{\sqrt{x+1}-\sqrt{x}}{\{\sqrt{x+1}+\sqrt{x}\}\{\sqrt{x+1}-\sqrt{x}\}}\,dx$

$= \int_0^1 \frac{\sqrt{x+1}-\sqrt{x}}{(\cancel{x}+1)-(\cancel{x})}\,dx = \int_0^1 (\sqrt{x+1}-\sqrt{x})\,dx$

$= \int_0^1 \{(x+1)^{1/2}-(x)^{1/2}\}\,dx$

$= \left[\frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2}\right]_0^1$

$= \left[\frac{2}{3}\cdot 2^{3/2} - \frac{2}{3}1^{3/2}\right] - \left[\frac{2}{3}1^{3/2} - 0\right]$

$= \frac{4\sqrt{2}}{3} - \frac{2}{3} - \frac{2}{3} = \frac{4\sqrt{2}}{3} - \frac{4}{3} = \frac{4}{3}(\sqrt{2}-1)$ **Ans**

Ques Evaluate $\int_0^{\pi/4} \tan^2\theta\,d\theta$

Soln $I = \int_0^{\pi/4} \tan^2\theta\,d\theta = \int_0^{\pi/4} (\sec^2\theta - 1)\,d\theta$

$= \left[\tan\theta - \theta\right]_0^{\pi/4} = \tan\frac{\pi}{4} - \frac{\pi}{4} - \tan 0 - 0$

$= 1 - \pi/4$

Ques Evaluate $\int_0^{\pi/2} \sin^3\theta \, d\theta$

Soln $I = \int_0^{\pi/2} \sin^3\theta \, d\theta = \frac{1}{4}\int_0^{\pi/2} 4\sin^3\theta \, d\theta$

$= \frac{1}{4}\int_0^{\pi/2} (3\sin\theta - \sin 3\theta) \, d\theta$

$= \frac{3}{4}\int_0^{\pi/2} \sin\theta \, d\theta - \frac{1}{4}\int \sin 3\theta \, d\theta$

$= \frac{3}{4}\left[-\cos\theta\right]_0^{\pi/2} - \frac{1}{4}\left[-\frac{\cos 3\theta}{3}\right]_0^{\pi/2}$

$= \frac{3}{4}\left[-\cos\frac{\pi}{2} + \cos 0\right] - \frac{1}{12}\left[\cos\frac{3\pi}{2} - \cos 0\right]$

$= \frac{3}{4}\left[0+1\right] - \frac{1}{12}\left[0+1\right]$

$= \frac{3}{4} - \frac{1}{12} = \frac{9-1}{12} = \frac{8}{12} = \frac{2}{3}$

Ans

Ques Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sin x}$

Soln $I = \int_0^{\pi/2} \frac{dx}{1+\sin x} dx$

$= \int_0^{\pi/2} \frac{(1-\sin x)}{1-\sin^2 x} dx$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(1-\sin x)\,dx}{\cos^2 x}$$

$$= \int_0^{\pi/2} (\sec^2 x - \sec x \tan x)\,dx$$

$$= \int_0^{\pi/2} \sec^2 x\,dx - \int_0^{\pi/2} \sec x \tan x\,dx$$

$$= \left[\tan x\right]_0^{\pi/2} - \left[\sec x\right]_0^{\pi/2}$$

$$= \tan\frac{\pi}{2} - \tan 0 - \sec\frac{\pi}{2} + \sec 0$$

$$= -0 + 1 = 1 \quad \left\{ \because \tan\tfrac{\pi}{2} \ \& \ \sec\tfrac{\pi}{2} \text{ are not defined} \right\}$$

Alternate Method

$$\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \int \frac{dx}{2\cos^2 \frac{1}{2}\left(\frac{\pi}{2}-x\right)} \quad \left(\because \cos x = 2\cos^2\frac{x}{2} - 1\right)$$

$$= \frac{1}{2}\int \frac{dx}{\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} = \frac{1}{2}\int \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$\therefore I = \frac{1}{2}\int_0^{\pi/2} \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx = \cancel{\frac{1}{2}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi/2}}$$

$$= \frac{\frac{1}{\cancel{2}}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi/2}}{-\cancel{1/2}} = -\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi/2}$$

$$= -\left[\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}-0\right)\right] = -\left[\tan 0 - \tan\pi/4\right]$$

$$= -(0-1) = 1$$

Ans